CALCULUS BC

6.2 Antidifferentiation by Substitution

Indefinite Integrals

Ex. 1 If $f(x) = x^3 + 1$ and $u=x^2$ find each of the following antiderivatives in terms of x.

(a) $\int f(x)dx$

$\int (x^3+1)dx$

$= \frac{x^4}{4} + x + c$

(b) $\int f(u)du$

$\int (u^3+1)du$

$= \frac{u^4}{4} + u + c$

$= \frac{x^8}{4} + x^2 + c$

(c) $\int f(u)dx$

$\int (u^3+1)dx$

$\int (x^6+1)dx$

$\frac{x^7}{7} + x + C$

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Substitution

A change of variable can often help turn an unfamiliar integral into one we can evaluate.

Ex. 1 Evaluate $\int \cos x e^{\sin x} dx$

Let $u = \sin x$
$du = \cos x\, dx$

$\int e^u du$

$e^u + c$

$= e^{\sin x} + c$

Ex. 2 Evaluate $\int \frac{x^2}{(x^3-8)^5} dx$

Let $u = x^3 - 8$
$du = 3x^2 dx$
$\frac{1}{3} du = x^2 dx$

$\frac{1}{3} \int \frac{du}{u^5}$

$-\frac{1}{3} \frac{1}{4} u^{-4} + c$

$-\frac{1}{12(x^3-8)^4} + c$

Ex. 3 Evaluate $\int \cot 7x\, dx$

$\int \frac{\cos 7x}{\sin 7x} dx$

Let $u = \sin 7x$
$du = 7\cos 7x\, dx$
$\frac{1}{7} du = \cos 7x\, dx$

$\frac{1}{7} \int \frac{du}{u}$

$\frac{1}{7} \ln|u| + c$

$\frac{1}{7} \ln|\sin 7x| + c$

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Ex. 4 Find the indefinite integrals, using trig identities to help in your substitution method.

(a) $\int \sin^3 x dx$ (b) $\int \sec^6 x dx$ (c) $\int \frac{\tan x}{\cos^2 x} dx$

$\int \sin x \sin^2 x dx$

$\int \sin x (1-\cos^2 x) dx$

Let $u = \cos x$

$du = -\sin x dx$

$-\int (1-u^2) du$

$-\left(u - \frac{u^3}{3}\right) + c$

$-\cos x + \frac{\cos^3 x}{3} + c$

b) $\int \sec^6 x dx$

$\int \sec^4 x \sec^2 x dx$

$\int \sec^4 x (1+\tan^2 x) dx$

$u = \tan x$

$du = \sec^2 x dx$

$\int \sec^2 x (1+u^2) du$

$\int (1+\tan^2 x)(1+u^2) du$

$\int (1+u^2)(1+u^2) du$

$\int (1+2u^2+u^4) du$

$u + \frac{2}{3}u^3 + \frac{1}{5}u^5 + c$

$\tan x + \frac{2}{3}\tan^3 x + \frac{1}{5}\tan^5 x + c$

c) $\int \frac{\tan x}{\cos^2 x} dx$

$\int \tan x \sec^2 x dx$

Let $u = \tan x$

$du = \sec^2 x dx$

$\int u du$

$\frac{u^2}{2} + c$

$\frac{\tan^2 x}{2} + c$

$\int \frac{\sin x}{\cos^3 x} dx$

let $u = \cos x$

$du = -\sin x dx$

$-\int \frac{du}{u^3}$

$\frac{1}{2u^2} + c$

$\frac{1}{2\cos^2 x} + c$

$\frac{1}{2}\sec^2 x + c$

$\frac{1}{2}(\tan^2 x + 1) + c$

$\frac{1}{2}\tan^2 x + \frac{1}{2} + c$

$\frac{1}{2}\tan^2 x + C_1$

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Ex. 5 Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sqrt{1+\cos x}}\,dx$

$u = 1+\cos x$
$du = -\sin x\,dx$
$-du = \sin x\,dx$
$u_u = 1+\cos\frac{\pi}{2} = 1$
$u_L = 1+\cos 0 = 2$

$$-\int_2^1 \frac{du}{\sqrt{u}}$$
$$\int_1^2 \frac{du}{\sqrt{u}}$$
$$2\sqrt{u}\Big]_1^2$$
$$= \underline{2\sqrt{2}-2}$$

Ex. 6 Evaluate $\int_0^1 \frac{x}{x^2-4}\,dx$

Let $u = x^2-4$
$du = 2x\,dx$
$\frac{1}{2}du = x\,dx$
$u_U = -3$
$u_L = -4$

$$\frac{1}{2}\int_{-4}^{-3} \frac{du}{u}$$
$$\frac{1}{2}\ln|u|\Big]_{-4}^{-3}$$
$$\frac{1}{2}\left[\ln|-3| - \ln|-4|\right]$$
$$= \frac{1}{2}\ln 3/4$$
$$= \underline{\ln\sqrt{3/4}} = \underline{\ln\left(\frac{\sqrt{3}}{2}\right)}$$